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AIRBLAST VULNERABILITY ENVELOPES FOR SUPERSONIC AEROSPACE VEHICLES

Gerald M. Campbell

MARCH 1979

Final Report

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A procedure for constructing airblast vulnerability envelopes for supersonic vehicles has been derived which enables the analyst to determine which portions of the envelope are valid. For simplicity, an overpressure of 1 lb/in^2 is used as a damage criterion throughout this report and nonhomogeneous atmosphere effects are ignored. Maximum and minimum shock front velocities are derived for the catching curve as well as the lower limit for the detonation orientation angle θ.

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PREFACE

The basis for this research was provided by previous work performed in collaboration with Mr R. E. Holladay of the Air Force Weapons Laboratory (IN), Kirtland Air Force Base, New Mexico.

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SECTION I

INTRODUCTION

Although sometimes technically difficult, the construction of nuclear blast vulnerability envelopes for vehicles operating at subsonic speeds is a perfectly straightforward procedure from a theoretical point of view. One merely determines an overpressure or gust velocity level which will provide the required vehicle damage. Then, for a specific yield and set of target operational parameters, detonation locations are computed, as a function of target and shock front velocities, which will subject the target to that damage level. The locus of all such detonations forms the vulnerability volume for the specified conditions and a two-dimensional slice through this volume containing the vehicle is a vulnerability envelope. In actual practice the blast parameter level necessary to produce a certain level of damage varies considerably with various target operational parameters and with the angle at which the shock front intercepts the target. These complications must be considered in the actual construction of vulnerability envelopes; however, they are unnecessary for the purpose of this report and are omitted. For simplicity, an overpressure of 6.9 kPa (1 lb/in^2) is used as a damage criterion throughout this report and nonhomogeneous atmosphere effects are ignored.

When this same procedure is used to construct blast vulnerability envelopes for supersonic vehicles, very strange results can frequently be obtained. In particular, it is not at all uncommon to find envelopes, such as those shown in figure 1, where a fairly low overpressure (~ 6.9 kPa) is used as the damage criterion. A check of these envelopes will show that detonations on the envelope will subject the target to the determined blast criterion.

Such envelopes are highly disturbing, since both logic and physics dictate that every vulnerability envelope should contain the target. They also must be incorrect, since it is obvious that detonations along the flight path between the target and the envelope would produce much greater overpressures on the target than would detonations within the envelope. This is inconsistent with any possible designation of the envelope: i.e., sure-safe, sure-kill, etc.

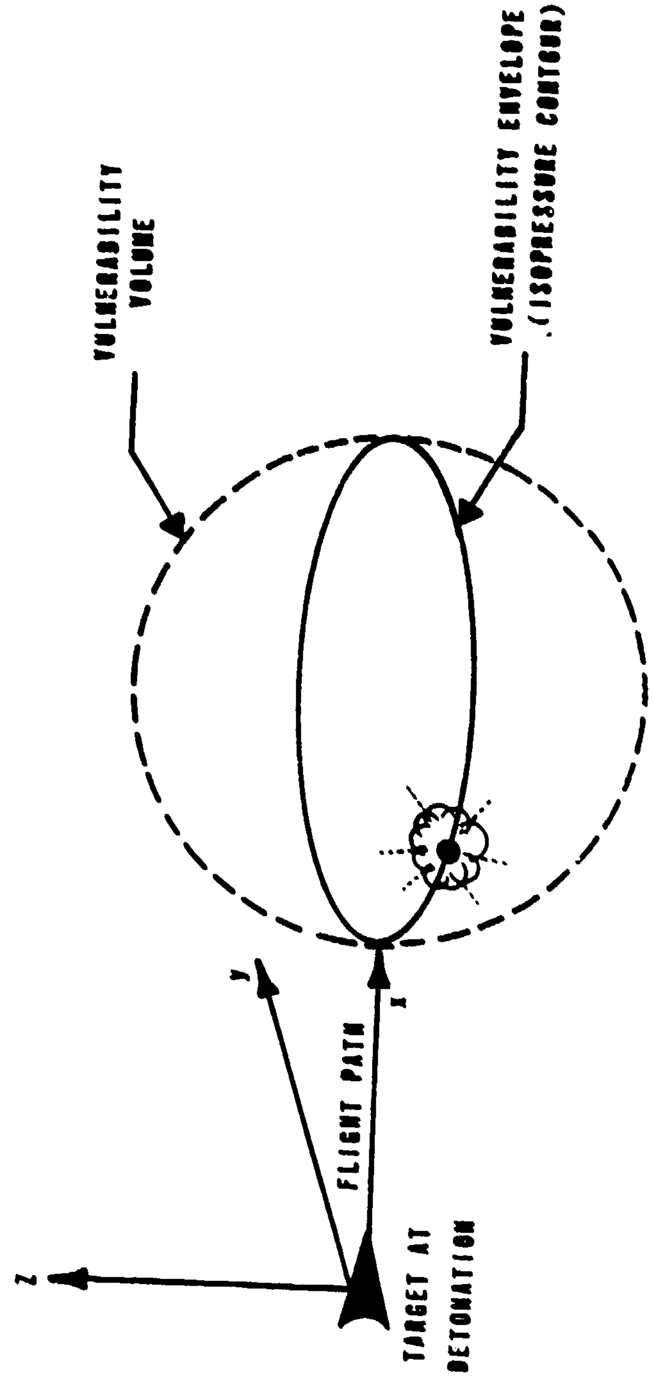

Figure 1. Erroneous Vulnerability Envelope

The problem of dealing with such envelopes is very real. It has not been at all uncommon in the past to find vulnerability studies containing blast vulnerability envelopes which either resemble the envelope in figure 1, or are not closed on the left side—an equally unsatisfactory condition.

The purpose of this report is to explain how such envelopes are obtained and why they are not satisfactory from a vulnerability standpoint, and to present a much more logical and consistent method for constructing vulnerability envelopes for supersonic vehicles.

SECTION II

BLAST VULNERABILITY ENVELOPES

MULTIPLE SHOCK INTERCEPTIONS

Blast vulnerability envelopes, similar to figure 1, for supersonic vehicles result from double valued solutions to shock interceptions similar to the solution shown in figure 2 where (r, θ) defines the detonation point with respect to a moving target. After detonation, the target travels a distance $R' = Vt'$ where t' is the time required for the shock front to travel a distance d' and intercept the target. Call this a frontside interception since the target meets the shock front head-on. After this interception, the shock front continues to expand radially with a supersonic velocity which decays asymptotically as it approaches Mach 1. Therefore, a supersonic vehicle may catch the shock front again after traveling a distance $R = Vt$, where t is the longer time required for the shock front to travel the somewhat greater distance d. Call this a backside interception since the vehicle overtakes the shock front from the rear.

If the overpressure at the backside interception meets the desired criterion, the analyst may use the associated values for (r, θ) to define a point on the envelope and overlook the much higher overpressure at the frontside interception. For example, a vehicle velocity of Mach 1.83 at 11,277.6 m (37,000 ft) and an overpressure criterion of 6.9 kPa will produce a point on a 1 MT envelope (r, θ) = (982 m, 0.5236 rad). This is a backside solution and the analyst may not be aware that the frontside interception imposed an overpressure of 579 kPa on the aircraft (figure 2). If 6.9 kPa were the sure-safe criterion, the envelope thus calculated would be the locus of points from which a 1 MT burst would impose 6.9 kPa on the vehicle, but it is not a sure-safe envelope.

Where both solutions exist, the overpressure associated with the frontside solution is always higher than that for the backside solution. Only the frontside solution is valid for use as a point on a vulnerability envelope. When envelopes such as that shown in figure 1 are constructed, the right side of the envelope will consist of frontside solutions and the left side (detonation points near the origin) will be made up of backside solutions. A method for identifying backside solutions is explained in the following paragraph. A solution for this portion of the vulnerability envelope will be discussed in section III.

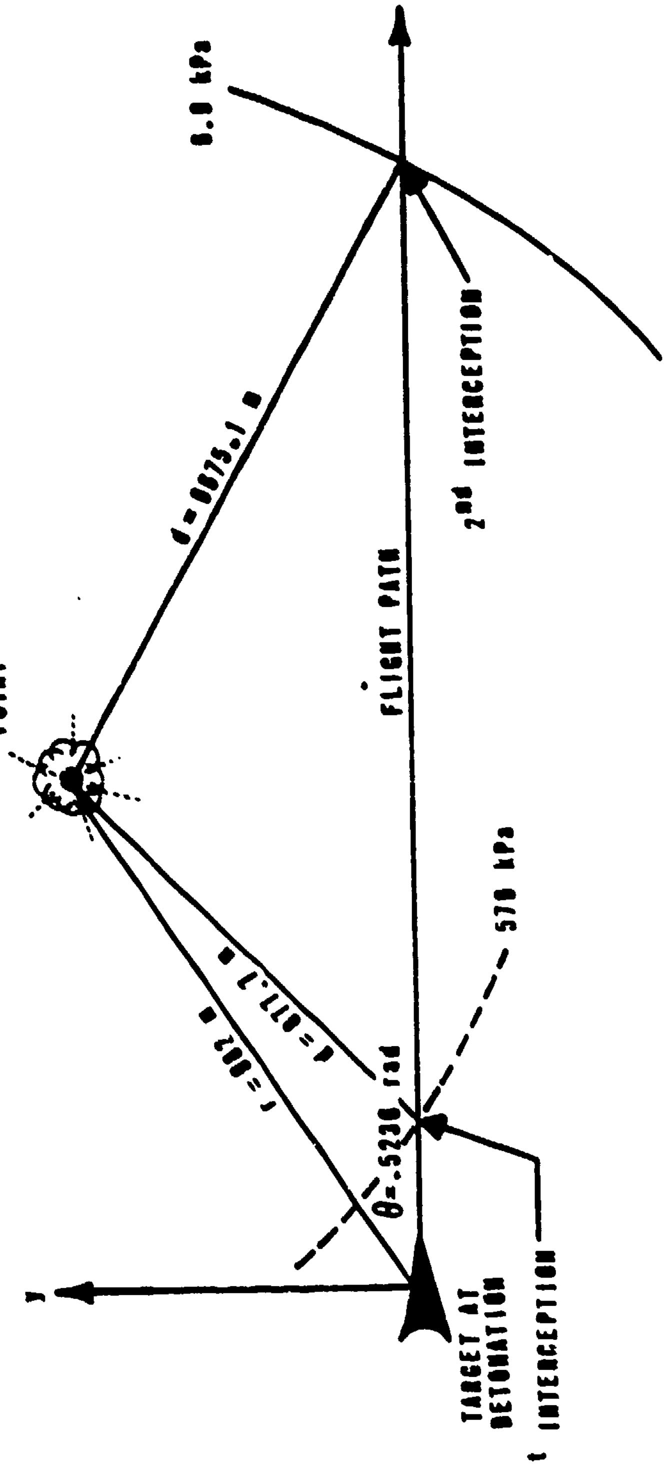

Figure 2. Geometry for Multiple Shock Interceptions

ENVELOPE CONSTRUCTION

The production, checking, or even discussion of blast vulnerability envelopes requires a source of information for shock front parameter values. A person constructing such envelopes should use a computer code such as the Air Force Weapons Laboratory (AFWL) Nuclear Blast Standard (ref. 1) for this information. However, for the purposes of this report it was more convenient merely to scale the 1 KT sea-level parameters from the IBM Problem-M Curves (ref. 2), using conventional scaling laws such as those given in reference 3 and reproduced in the appendix for convenience. All examples presented in this report are based on a 1 MT weapon detonated at 11,277.6 m. Blast parameter values for this situation are shown in figure 3 along with the average shock velocity $\bar{U} = d/t$ which was computed directly using the shock arrival time curve.

This geometry, applicable to the blast envelope calculation, is shown in figure 4. The range, from the detonation point at second intercept, may be represented by

$$d^2 = r^2 + R^2 - 2\ rR\ \cos\theta$$

Substituting Vt for R, and solving for the slant range at detonation r, gives

$$r = Vt\ \cos\theta \pm \left(d^2 - V^2\ t^2\ \sin^2\theta\right)^{1/2} \qquad (1)$$

The target velocity, V, is known and d and t are determined by the envelope criterion. Therefore, the envelope construction reduces to selecting appropriate values for θ, solving for the associated slant ranges r, and plotting the values for (r, θ) as points on the envelope. Unfortunately, except for $r = Vt \cos\theta$, the real solutions of equation (1) come in pairs. The shorter range of each pair is always a backside solution if it has the same sign as the larger solution (both positive or both negative). A positive indication that an envelope such as that shown in figure 1 is being constructed is the existence of an angle θ such that the two solutions for equation (1) have the same sign. Another sure indication is the existence of an angle θ such that equation (1) has complex solutions.

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1. Needham, C. E., et al., *Nuclear Blast Standard (1 KT)*, AFWL-TR-73-55 (Rev), Air Force Weapons Laboratory, Kirtland AFB, NM, 1975.
2. Broyles, C. D., *IBM Problem-M Curves*, SCTM 268-56-51, Los Alamos Scientific Laboratory, Los Alamos, NM, 1956.
3. Glasstone, S., and Dolan P. J., *The Effects of Nuclear Weapons* (3rd Ed), United States Department of Defense, Wash D C, 1977.

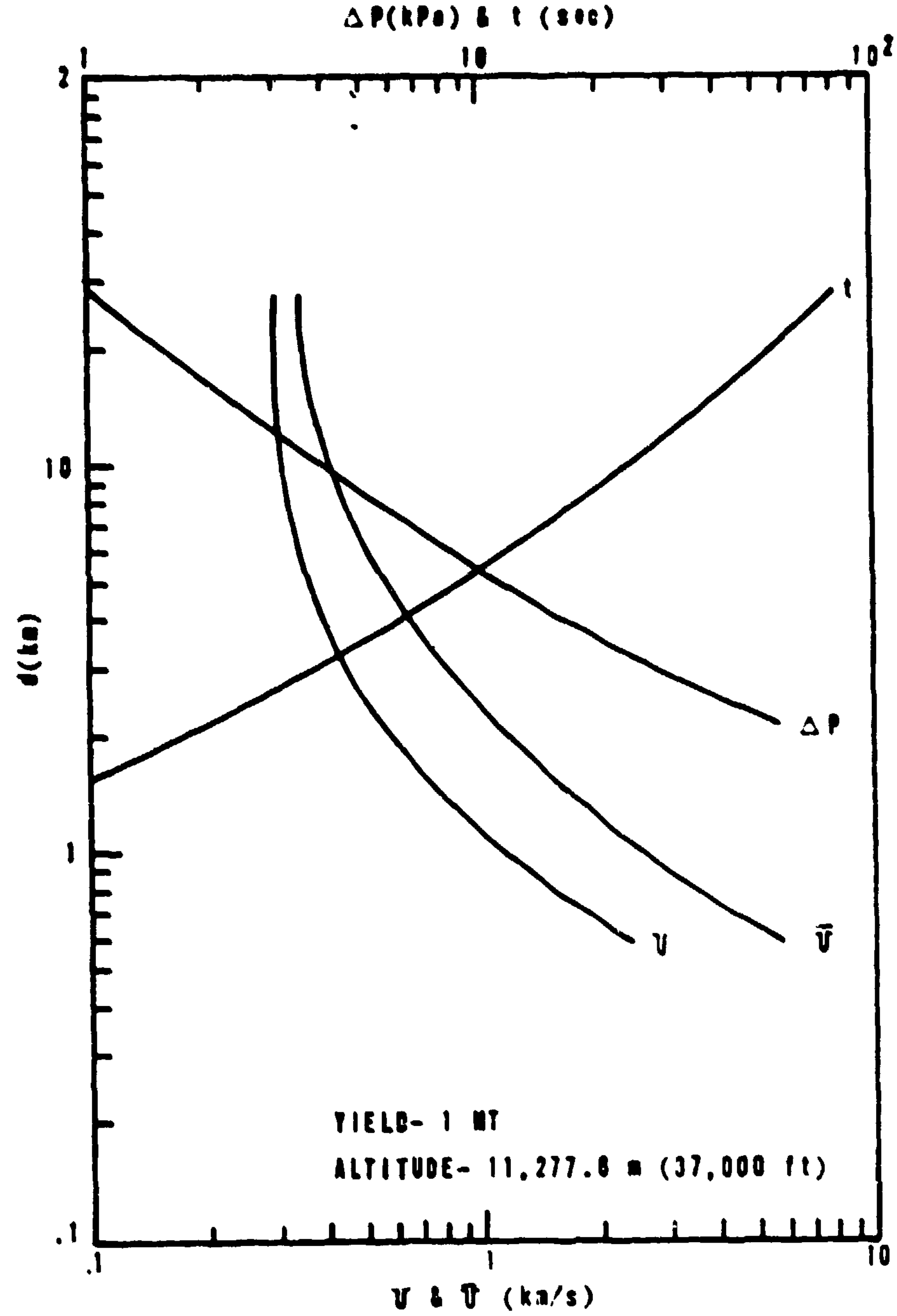

Figure 3. Nuclear Blast Parameters

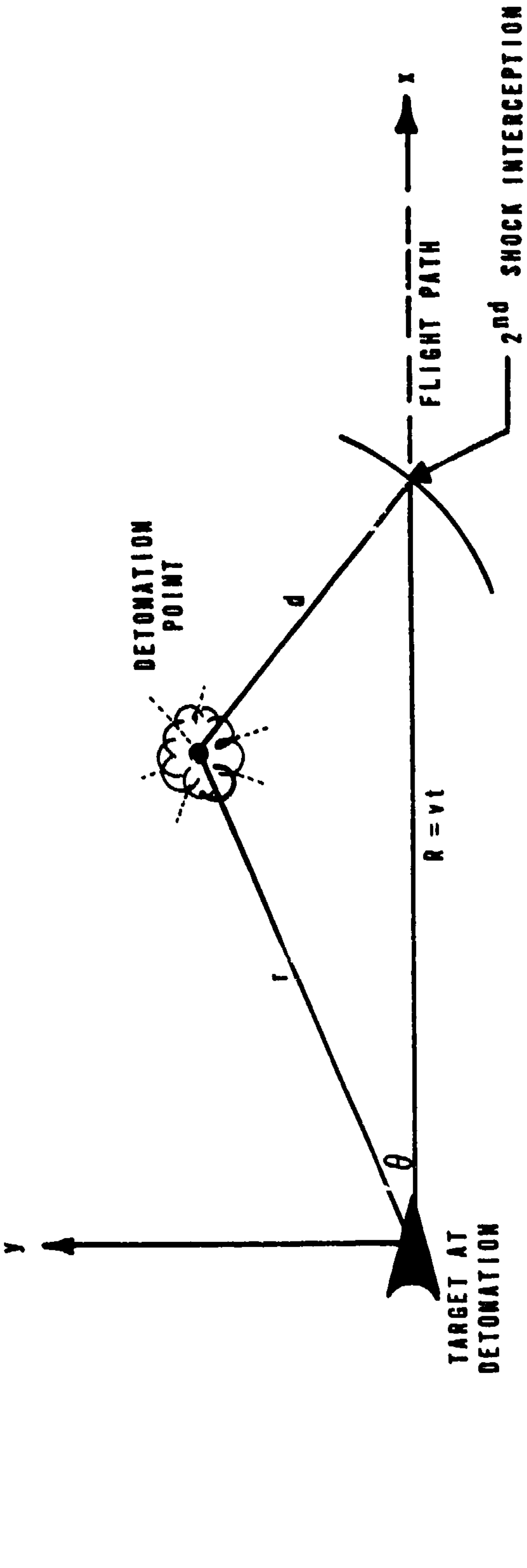

Figure 4. Blast Vulnerability Envelope Geometry

These two conditions are equivalent because they exist only if the envelope blast criterion corresponds to an average shock velocity less than the target velocity.

This is an extremely important point. If the blast overpressure (used as the vulnerability envelope criterion) corresponds to an average shock velocity less than the target velocity, then the envelope cannot be logically constructed entirely from equation (1). For example, suppose an aircraft is vulnerable to 6.9 kPa from a 1 MT detonation at 11,277.6 m. From figure 3, the average shock velocity corresponding to this criterion is 490.7 m/s or less.

Therefore, equation (1) may be used to construct vulnerability envelopes for aircraft velocities of 490.7 m/s or less. A method is outlined in the next section for the construction of envelopes applicable to higher aircraft velocities.

Although equation (1) can be misused, it is still a valid equation describing a definite physical situation. Therefore, if a complex range value is ever obtained, it will be true that the angle θ denotes a direction from which the given weapon cannot impose the blast criterion on the target regardless of how close or how far from the target it is detonated. This statement may appear contradictory, but it is fact and forms the basis for the corrective procedure suggested later.

Envelopes such as that shown in figure 1 result when the target velocity is greater than the average shock velocity corresponding to the overpressure criterion for the envelope and every real-valued range obtained from equation (1) is plotted. In this case, those points which define the envelope near the origin are not valid because they represent a backside solution to the problem.

VALID SOLUTIONS

The question arises as to which points on the vulnerability envelope are valid. In the simple example used here with a constant overpressure criterion, the answer is easy. Use only the larger of the two range values obtained for angles such that $|\sin\theta| \leq d/Vt$. However, in most actual situations the vulnerability criterion varies with θ so that this answer is not so easy to determine. Also, even if a point is known to represent a backside solution, it is frequently desirable to know the overpressure when the target first intercepts the shock front.

To investigate a point $\left(r_s, \theta\right)$ on a computed curve, it is necessary to graph both $d = \left(r_3^2 + V^2t^2 - 2r_sVt\cos\theta\right)^{1/2}$ and the d = f(t) curve which can be scaled from a 1 KT sea-level curve for time of shock arrival. The intersection of these curves gives both the front- and back-side solutions to the problem. The actual computation of the range d is easier if the equation is solved for

$$t = \left[r_s \cos\theta \pm \left(d^2 - r_s^2 \sin^2\theta\right)^{1/2}\right] \frac{1}{V}$$

and values of t tabulated as a function of d.

Table 1 shows two such tabulations made to check two points which were taken from an actual sure-safe envelope computed for a 1 MT weapon used against an aircraft flying at Mach 1.83 at an altitude of 11,277.6 m. Points were checked for the following cases:

<u>Case 1</u>

Criterion: ΔP = 4,34 kPa

Point: $\left(r_s, \theta\right)$ = (4998.7 m, 0.9215 rad)

<u>Case 2</u>

Criterion: ΔP = 2.9 kPa

Point: $\left(r_s, \theta\right)$ = (7924,8 m 0.8727 rad)

The values tabulated in table 1 are plotted in figure 5 with the range function, d = f(t), taken from figure 3. The overpressures associated with the intersection points were read from figure 3. They indicate that the two points investigated are backside solutions and that overpressures of 16 kPa and 7.8 kPa, respectively, would be imposed on the aircraft at its initial interceptions with the shock front from a detonation at these points. These overpressures are far higher than the backside overpressures of 4.34 kPa and 2.9 kPa, respectively, which meet the envelope criteria. The data from figure 5 indicate the fallacy in the sure-safe designation of the original envelope from which the points were taken.

Table 1

SLANT RANGE VERSUS SHOCK ARRIVAL TIME FOR TWO POINTS ON SURE-SAFE ENVELOPE

CASE 1

$(r,\theta) = (4998.7 \text{ m}, 0.9215 \text{ rad})$

d (km)	t(s)	
3.99	5.6	
4.11	3.67	7.52
4.27	2.75	8.44
4.57	1.44	9.76
5.18	--	11.7
6.1	--	14.1
9.14	--	20.8
10.67	--	23.9
15.24	--	32.8

$$t = 1/.5401 \left[3.022 \pm \left(d^2 - 15.853\right)^{1/2} \right]$$

CASE 2

$(r, \theta) = (7924.8 \text{ m}, 0.8727 \text{ rad})$

d (km)	t(s)	
6.07	9.43	
6.1	8.41	10.5
6.4	5.68	13.2
6.71	4.16	14.7
7.01	2.94	15.9
7.62	--	18.0
8.23	--	19.7
9.14	--	22.1
10.36	--	24.98
12.19	--	29.85
18.29	--	41.4

$$t = 1/.5401 \left[5.094 \pm \left(d^2 - 36.854\right)^{1/2} \right]$$

Yield = 1 MT Velocity = 540.1 m/s Altitude = 11,277.6 m

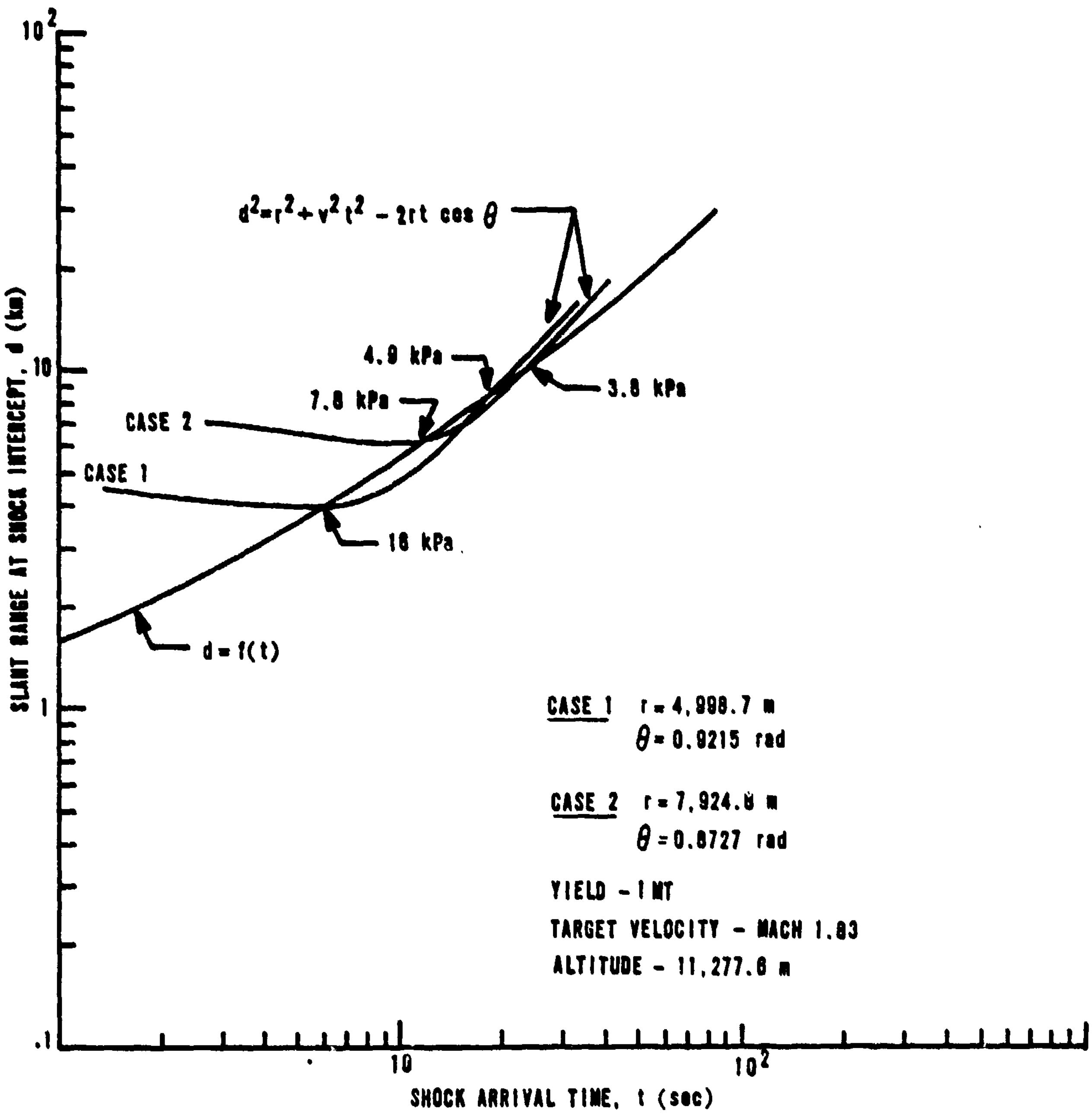

Figure 5. Backside versus Frontside Solutions

SECTION III

CATCHING CURVE

GENERAL DESCRIPTION

The basic difficulties described in the previous sections arise from the fact that the shock front velocity, initally much greater than Mach 1, asymptotically approaches Mach 1. Therefore, in the case of supersonic targets, the target can outrun the shock wave for certain detonation orientations. For a given altitude, yield, and target velocity, a curve can be drawn about the target such that the shock front will intercept the target for detonations inside the curve and will not intercept the aircraft for detonations outside the curve. This curve will be called the catching curve since it is the locus of detonation points from which the shock front will just catch the aircraft. The calculations of the catching curve will be discussed in this section. In general, the curve will appear as shown in figure 6.

The equations derived in this section indicate that, as the slant range increases, the equation for the catching curve approaches $\sin\theta = 1/M$ where M defines target Mach number. However, there is normally no advantage in extending the curve for long ranges. Figure 6 is based on the assumption that the target is flying straight and level. For other flight paths, the catching curve would look somewhat different.

The catching curve portion of interest is located near the origin, where very large discontinuities exist in the overpressures which can be imposed upon the target. For example, if the target velocity and altitude are 515.1 m/s and 11,277.6 m, respectively, a 1 MT detonation on the catching curve directly behind the target will impose 41.6 kPa on the target. The same detonation just outside the catching curve will not impose any overpressure on the target because the shock front cannot catch it. Thus, in this situation the minimum overpressure which can be imposed on the target from behind is 41.6 kPa. By applying the criterion that the average shock velocity must be greater than the target velocity, it can be determined from figure 3 that it is not feasible to construct a 6.9 kPa envelope for this case; i.e., an overpressure of 6.9 kPa corresponds to an average shock front velocity of 490.7 m/s, which is less than the target velocity of 515.1 m/s.

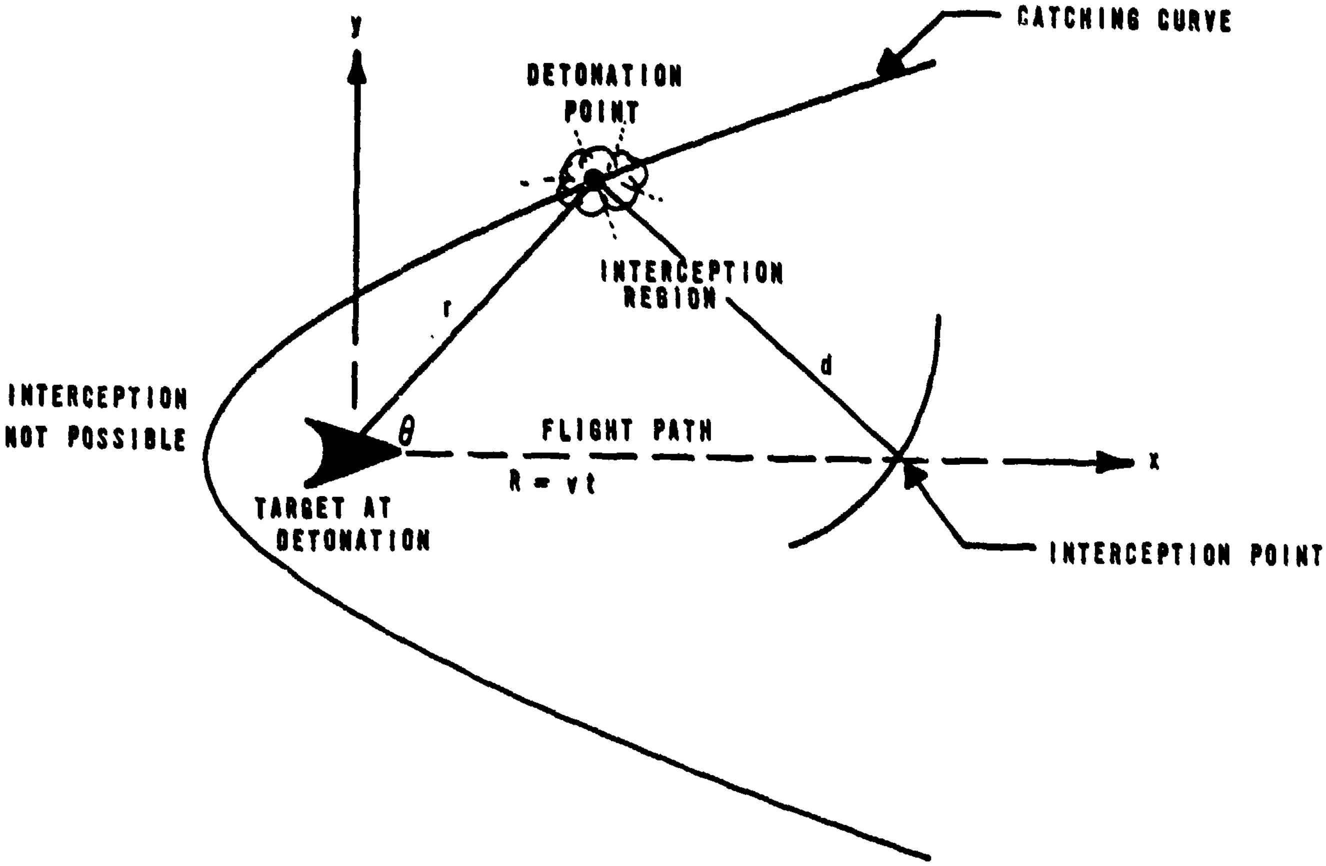

Figure 6. Catching Curve for Supersonic Targets

When confronted with these situations, the only logical recourse is to redefine the envelope criterion for bursts near the target. This may be done by using the catching curve as the left side of the envelope (for targets flying to the right). This will, in general, produce envelopes such as the sure-safe and sure-kill ones shown in figure 7.

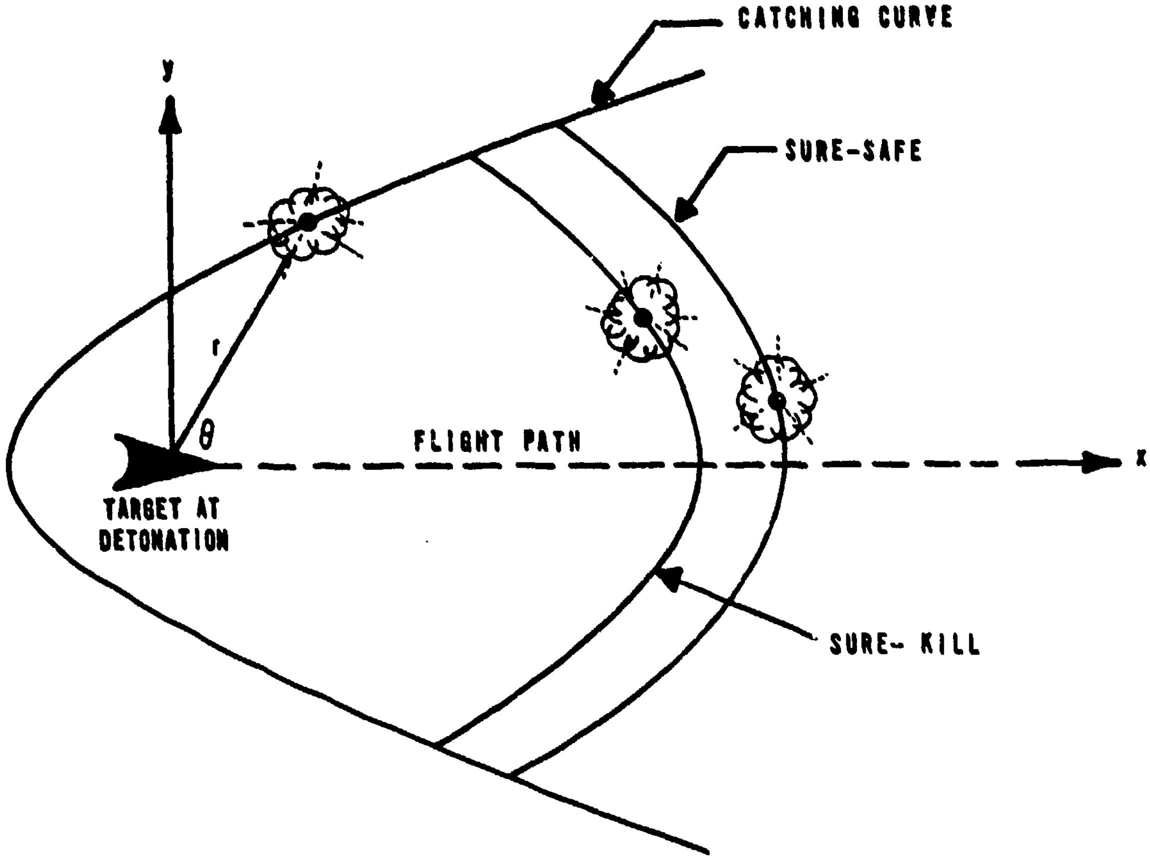

Figure 7. Sure-Safe and Sure-Kill Envelopes for Supersonic Targets

At the points where an envelope intercepts the catching curve, the original envelope criteria are abandoned and the catching curve becomes the envelope. Such an envelope always contains the target since the initial shock front velocity is much greater than the target velocity. In general, detonations on the catching curve near the target will impose overpressure on the target much higher than that required to kill it.

Strangely, a portion of the catching curve can represent both the sure-safe and sure-kill envelopes. It is a sure-safe curve since the shock front from a detonation outside of it will damage the target catastrophically. Similar logic will allow the left portion of the catching curve to serve as an extension of a blast parameter value curve or of a particular damage level curve.

DERIVATION

To derive a formula for the catching curve, it is necessary to use equation (1) as derived from the target interception geometry depicted in figure 8. For this particular case, d is no longer defined by an envelope criterion. Since d is a function of time, this equation for constant θ expresses r as a function of possible arrival times that will allow the shock front to intercept the target. The greatest possible slant range r gives a point (r, θ) on the catching curve and may be located by differentiating equation (1) with respect to t and setting $\partial r/\partial t = 0$. Thus

$$\frac{\partial r}{\partial t} = 0 = V \cos\theta \pm \frac{d\partial d}{\partial t} - V^2t^2 \sin^2\theta \left(d^2 - V^2t^2 \sin\theta \right)^{1/2}$$

By substituting U for the shock front velocity $\partial d/\partial t$ and rearranging, the catching curve equation can be written implicitly as

$$\sin^2\theta = \frac{1 - (U/V)^2}{1 + (V/\bar{U}) - 2(U/\bar{U})} \qquad (2)$$

where $\bar{U}$ represents the average shock velocity d/t.

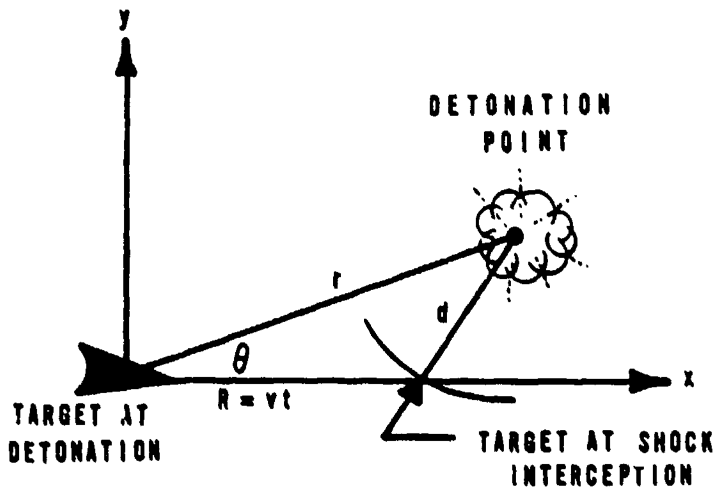

Figure 8. Geometry for Shock Front Interception of Target

A point (r, θ) on the catching curve can most easily be determined by selecting a shock velocity U which will also determine values for $\overline{U}$, d, and t. Then U and $\overline{U}$ can be inserted in equation (2) to solve for θ and then use θ, d, and t in equation (1) to solve for r. A proper selection of U will always yield a real positive value for r, and this is the solution which should be used.

The maximum value of U associated with a point on the catching curve may be found by differentiating equation (2) with respect to θ and setting $\partial U/\partial\theta = \partial\overline{U}/\partial\theta = 0$ (since U and $\overline{U}$ will achieve a maximum at the same angle). When this is done every term on the right-hand side will contain either $\partial U/\partial\theta$ or $\partial\overline{U}/\partial\theta$ leaving $\sin\theta \cos\theta = 0$. Therefore, the maximum shock velocity occurs for $\theta = \pm\,\pi/2$ or $\theta = \pi$. At $\theta = \pi$, equation (2) readily yields $U = V$ and for $\theta = \pm\,\pi/2$ we have $U = V^2/\overline{U}$.

Now $\overline{U} > V$ is necessary if the shock front is to catch the target, so $V > V^2/\overline{U}$ indicates that the maximum shock velocity associated with the catching curve occurs for $\theta = \pi$ (detonation directly behind the target). This, of course, is the expected result. The minimum shock velocity which can be chosen is $U = C$ = ambient sonic velocity. Therefore, in solving the catching curve equation (2), the selection rule for the shock velocity is $C < U \leq V$.

The above discussion contains several useful pieces of information. Chief among them is a method for resolving an ambiguity in the catching curve equation. Since $\sin(\pi/2 + \phi) = \sin(\pi/2 - \phi)$ for $0 \leq \phi \leq \pi/2$, it is frequently difficult to select the proper value of θ when solving the formula. The problem always exists in computerized solutions and can even cause trouble during a hand solution if the angle is near 90°. This ambiguity can be easily resolved if the shock velocity associated with $\theta = \pi/2$ is first determined. That is, determine U_1 such that $U_1\overline{U}_1 = V^2$. Then $U_1 \leq U \leq V$ implies $\pm\,\pi/2 < \theta < \pm\,\pi$ and $U \leq U_1$ implies $\theta < \pi/2$.

Another item of information is the minimum overpressures which can be imposed on the target from a detonation directly behind it. This is the overpressure associated with a shock front velocity which is equal to the target velocity.

The lower limit for the angle θ in the catching curve equation can be found by setting $U = \overline{U} = C$.

Thus

$$\sin^2\theta = \left(\frac{C}{V}\right)^2$$

and the lower angular limit is $\theta = \sin^{-1}(1/M)$ where M is the target velocity expressed as a Mach number. The catching curve equation (2) has the inherent advantage of being directly applicable for any altitude and weapon yield provided that U, $\overline{U}$, and V are expressed as Mach numbers. However, an examination of equation (1) shows that the slant range r, associated with the catching curve for a 1 KT detonation at sea level, must be scaled for other weapon yields and altitudes of interest, i.e.,

$$r = r_o\left(\frac{W}{P/P_o}\right)^{1/3}$$

Where the subscript zero denotes sea-level conditions and

W = weapon yield (kilotons)

r = slant range from detonation point to target (meters)

P/P_o = ratio of ambient atmospheric pressure to that at sea level

APPLICATION

The example in this section demonstrates the use of equation (2) in the construction of a catching curve for the following hypothetical set of conditions:

Weapon yield	- 1 MT
Detonation Altitude	- 11,277.6 m
Aircraft Velocity	- Mach 1.83
Aircraft Altitude	- 11,277.6 m

Table 2 summarizes the computations for a number of detonation points (r, θ) on the catching curve for selected shock front velocities and corresponding overpressure levels. These parameters were then used to construct the catching curve shown in figure 9.

Table 2

CATCHING CURVE PARAMETERS FOR SUPERSONIC TARGET

d(km)	t (sec)	U (km/s)	$\bar{U}$ (km/s)	$\sin^2\theta$	$\cos\theta$	θ (rad)	r (km)	ΔP (kPa)
2.19	1.98	0.54	1.1	0	-1	π	1.12	55.9
2.26	2.12	0.52	1.07	0.221	$\bar{0}.883$	2.6529	1.19	51.7
2.37	2.31	0.51	1.02	0.408	$\bar{0}.769$	2.4435	1.27	47.6
2.67	2.89	0.46	0.93	0.793	$\bar{0}.455$	2.0595	1.57	36.5
2.93	3.47	0.44	0.84	0.913	$\bar{0}.295$	1.8850	1.86	30.3
3.1	3.85	0.43	0.81	0.961	$\bar{0}.197$	1.7803	1.93	26.9
3.17	4.05	0.41	0.79	0.974	$\bar{0}.161$	1.7418	1.99	25.5
3.38	4.53	0.42	0.75	1	0	$\pi/2$	2.34	22.1
3.54	4.82	0.40	0.74	1	0.017	1.5533	2.45	20
3.93	5.78	0.39	0.68	0.987	0.114	1.4556	2.75	16.6
5.33	9.63	0.35	0.55	0.850	0.387	1.1746	4.39	9.7
6.68	13.5	0.34	0.49	0.734	0.516	1.0280	6.09	6.9
8.59	19.3	0.33	0.45	0.633	0.606	0.9198	8.66	4.7
14.78	38.5	0.31	0.38	0.494	0.711	0.7802	16.82	2.3
20.88	57.8	0.30	0.36	0.443	0.746	0.7295	25.3	1.5

Yield = 1 MT   Velocity = 540.1 m/s   Altitude = 11,277.6 m

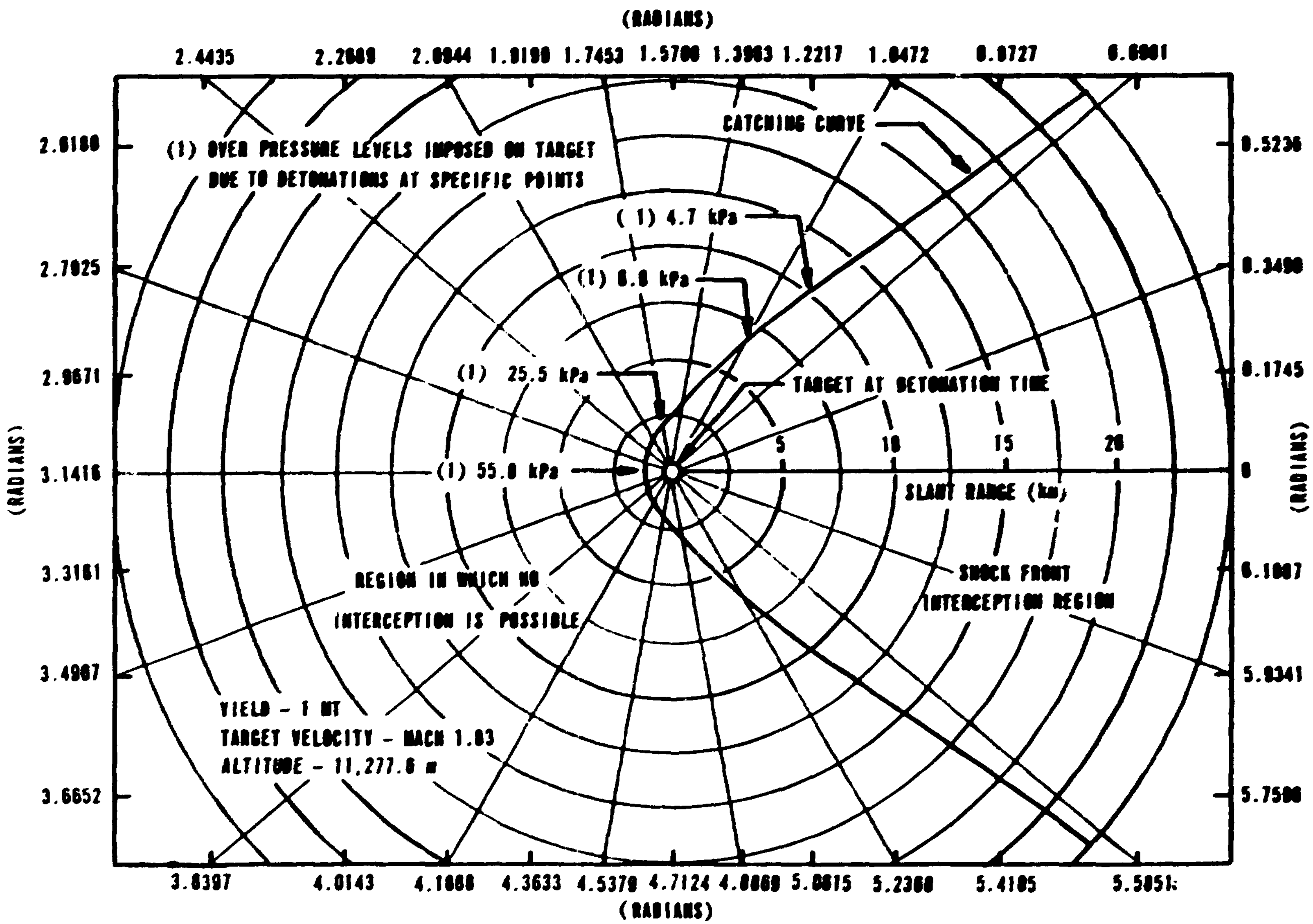

Figure 9. Catching Curve for Supersonic Target

SECTION IV

CONCLUSIONS

A method was developed for the purpose of demonstrating that it is possible for a supersonic vehicle to intercept the shock front from a nuclear detonation twice, incurring a substantially different level of overpressure at each intercept.

Using this technique, the locus of detonation points can be established from which nuclear shock fronts will just catch a target traveling at supersonic speeds. The contour formed by these points can be used to complete partially defined overpressure vulnerability envelopes associated with nuclear hardness assessments of supersonic aerospace vehicles.

The overpressure along such a contour varies as a function of slant range and orientation relative to the vehicle for a specific airspeed, weapon yield, and detonation altitude.

In general, detonations on the contour (catching curve) near the target will impose overpressures on the target much higher than that required to destroy it.

The catching curve partially represents both the sure-safe and sure-kill vulnerability envelopes because the shock front from a detonation which occurs outside the curve cannot catch the target and a detonation in the area bounded by the curve will damage the target catastrophically.

APPENDIX

CATCHING CURVE COMPUTATIONS

CONVENTIONAL SCALING LAWS

These scaling laws were used in the catching curve computations to calculate shock front parameters for weapon yields other than 1 kT and detonation altitudes other than sea level.

$$d = d_o \left(\frac{W}{P/P_o}\right)^{1/3}$$

$$\Delta P = \Delta P_o \left(P/P_o\right)$$

$$t = \frac{t_o}{C/C_o} \left(\frac{W}{P/P_o}\right)^{1/2}$$

$$U = U_o \left(C/C_o\right)$$

Where the subscript zero denotes sea level conditions and

W = weapon yield (KT)

d = slant range from detonation point to shock front (km)

ΔP = peak overpressure (kPa)

t = time of shock front arrival (s)

U = shock front velocity (km/s)

P/P_o = ratio of ambient atmospheric pressure to that at sea level

C/C_o = ratio of ambient speed of sound so that sea level

CATCHING CURVE DERIVATION

From the geometry of figure 8

$$r = Vt \cos\theta \pm \left(d^2 - V^2t^2 \sin^2\theta\right)^{1/2}$$

Thus

$$\frac{\partial r}{\partial t} = 0 = V \cos\theta \pm \left(d \frac{\partial d}{\partial t} - V^2 t \sin^2\theta\right) \Big/ \left(d^2 - V^2t^2 \sin^2\theta\right)^{1/2}$$

$$d^2 - V^2t^2 \sin^2\theta = \left(d^2U^2 + V^4t^2 \sin^4\theta - 2dUV^2t \sin^2\theta\right)/V^2 \cos^2\theta$$

where $U = \partial d/\partial t$ = shock front velocity. Thus

$$d^2V^2\cos^2\theta - V^4t^2\sin^2\theta\cos^2\theta = d^2U^2 + V^4t^2\sin^4\theta - 2dUV^2t\sin^2\theta$$

$$d^2V^2\cos^2\theta - d^2U^2 = V^4t^2\sin^2\theta - 2dUV^2t\sin^2\theta$$

$$\cos^2\theta - \frac{U^2}{V^2} = \frac{V^2}{\overline{U}^2}\sin^2\theta - 2\frac{U}{\overline{U}}\sin^2\theta = \left(\frac{V^2}{\overline{U}^2} - 2\frac{U}{\overline{U}}\right)\sin^2\theta$$

where $\overline{U} = d/t$ = average shock front velocity. Thus

$$1 - \sin^2\theta = \frac{U^2}{V^2} + \left(\frac{V^2}{\overline{U}^2} - 2\frac{U}{\overline{U}}\right)\sin^2\theta$$

$$1 - \frac{U^2}{V^2} = \left(1 + \frac{V^2}{\overline{U}^2} - 2\frac{U}{\overline{U}}\right)\sin^2\theta$$

so that the catching curve equation can be written implicitly as

$$\sin^2\theta = \frac{1 - (U/V)^2}{1 + (V/\overline{U})^2 - 2\,(U/\overline{U})}$$

or

$$\theta = \sin^{-1}\left(\frac{1 - (U/V)^2}{1 + (V/\overline{U})^2 - 2\,(U/\overline{U})}\right)^{1/2}$$

MAXIMUM SHOCK FRONT VELOCITY

The maximum shock front velocity U associated with a point on the catching curve may be found by differentiating the above equation with respect to θ and setting $\partial U/\partial\theta = \partial\overline{U}/\partial\theta = 0$ (since U and $\overline{U}$ will achieve a maximum at the same angle). When this is done every term on the right-hand side will contain either $\partial U/\partial\theta$ or $\partial\overline{U}/\partial\theta$ leaving $\sin\theta\cos\theta = 0$.

Therefore, the maximum shock velocity occurs for $\theta = \pm\,\pi/2$ or $\theta = \pi$. At $\theta = \pi$, the above equation readily yields $U = V$. For $\theta = \pm\,\pi/2$ we have

$$1 + (V/\overline{U})^2 - 2(U/\overline{U}) = 1 - (U/V)^2$$

$$V^4 - 2V^2U\overline{U} + U^2\overline{U}^2 = 0$$

$$(V^2 - U\overline{U})^2 = 0$$

so that $U = V^2/\overline{U}$. Now $\overline{U} > V$ is necessary if the shock front is to catch the target. Therefore, $V > V^2/\overline{U}$ means that the maximum shock velocity associated with the catching curve occurs for a detonation directly behind the target, i.e., $\theta = \pi$.

LOWER LIMIT FOR CATCHING CURVE ANGLE θ

The lower limit for the angle θ in the catching curve equation can be established by setting both the average shock front velocity $\overline{U}$ and the shock front velocity U equal to the ambient speed of sound C.
Thus

$$\sin^2\theta = \frac{1 - (C/V)^2}{(V/C)^2 - 1} = (C/V)^2$$

and the lower limit is $\theta = \sin^{-1}(1/M)$ where M represents the target velocity expressed as a Mach number.

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